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Allan Struthers, Ph.D. in Mathematics, May 19, 1991
Jose Matias, currently pursuing a Ph.D.

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Brief Outline of Research Findings

Work continued on the thermodynamics of two–phase continua. The general two–phase Stefan problem with supercooling, superheating, and capillarity, was studied [T19] in collaboration with M. Soner (Carnegie Mellon). Simple solutions – illustrating the chief differences between this problem and the classical Stefan problem – were obtained for the spherically symmetric problem, under the assumption of fast diffusion, with the liquid supercooled at infinity. It is shown that: (i) for $\Omega = \mathbb{R}^3$, a ball of the solid phase of sufficiently small size disappears in finite time, but a sufficiently large ball grows without bound; (ii) for $\Omega = \mathbb{R}^3$ and the solid phase initially situated in a spherical shell of thickness ϵ, the thickness of the solid shell initially increases, but the inner radius of this region decreases to zero in finite time T; the solid ball remaining at time T disappears at a later time or grows without bound according as ϵ is less than or greater than a critical value; in the limit $\epsilon \to 0$ the region occupied by the solid disappears infinitely fast; the problem has no solution for $\epsilon = 0$; (iii) when Ω is the region exterior to a sphere of radius R, with the boundary $r = R$ insulated and with the solid phase initially in a spherical shell of zero thickness at $r = R$, the solid phase grows without bound provided R is sufficiently large. While (ii) and (iii) are of little practical interest, they demonstrate the possibility of growth from a seed of zero volume.

Papers and reports which appeared under this contract

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[T2]* Gurtin, M.E., On a nonequilibrium thermodynamics of capillarity and phase, *Quarterly of Applied Mathematics*, **97**, 129–145 (1989).

[T3]* Gurtin, M.E., On diffusion in two–phase systems: the sharp interface versus the smooth transition layer, *Proceedings CNRS–NSF Symposium*, Nice (1988), Springer Lecture Notes in Physics **344**, (1989).

[T4]* Browning, R.V., M.E. Gurtin, and W.O. Williams, A model for viscoplastic materials with temperature dependence, *International Journal of Solids and Structures*, **25**, 441–457 (1989).

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[T6]* Milic, N., On non–equilibrium phase transitions in mixtures with interfacial structure, Ph.D. Thesis, Department of Mathematics, Carnegie Mellon University (December 1988).

[T7]* Gurtin, M.E., On the isothermal motion of a phase interface, *Proceedings of the Conference* (1988) "*Problems Involving Change of Type*", *Lecture Notes in Physics*, **359** (ed. K. Kirchgassner) Springer–Verlag, Berlin, 1990. .

[T8]* Gurtin, M.E., A. Struthers, and W.O. Williams, A transport theorem for moving interfaces, *Quarterly of Applied Mathematics*, **47**, 773–777 (1989).

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[T11]* Gurtin, M.E. and P. Podio–Guidugli, A hyperbolic theory for the evolution of plane curves, *SIAM Journal on Mathematical Analysis*, **22**, 575–586 (1991).

[T12]* Gurtin, M.E. and W.J. Hrusa, On the thermodynamics of viscoelastic materials of single–integral type, *Quarterly of Applied Mathematics*. Forthcoming.

[T13]* Gurtin, M.E., On the continuum mechanics of the motion of a phase interface. *Proceedings Seventh Army Conference on Applied Mathematics and Computing*, West Point (1989).

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[T15]* M.E. Gurtin and W.J. Hrusa, Global existence in one–dimensional nonlinear viscoelasticity with heat conduction, *Journal of Integral Equations*, 2, 431–460 (1990).

[T16]* F. Davi and M.E. Gurtin, On the motion of a phase interface by surface diffusion, *Zeitschrift für angewandte Mathematik und Physik*, 41, 782–811 (1990).

[T17]* M.E. Gurtin, On thermomechanical laws for the motion of a phase interface, *Zeitschrift für angewandte Mathematik und Physik*. Forthcoming.

[T18]* Gurtin, M.E., Evolving phase boundaries in the presence of deformation and surface stress, *Proceedings IMA Workshop on Evolving Phase Boundaries*, U. Minnesota (1990).

[T19] Gurtin, M.E. and H.M. Soner, Some remarks on the Stefan problem with surface structure, *Quarterly of Applied Mathematics*. Forthcoming.

*Discussed in previous progress reports.

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This paper discusses a generalized Stefan problem which allows for supercooling and superheating and for capillarity in the interface between phases. Simple solutions are obtained indicating the chief differences between this problem and the classical Stefan problem. A weak formulation of the general problem is given.

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